1. Details of module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 01 (Physics Part-1, Class XI)
Module Name/Title	Unit 2, Module 8**, Circular motion** Chapter 4, Motion in a plane
Module Id	Keph_10403_eContent
Pre-requisites	Speed, Acceleration, uniform motion, non-uniform motion , motion in two dimension ,motion in a plane, vector algebra
Objectives	**After going through this module, the learners will be able to**: • Understand Circular motion • Conceptualize Uniform circular motion Constant speed yet accelerating • Derive the expression for $a = v^2/r$ • Recognize the direction of centripetal acceleration • Distinguish between uniform and non-uniform circular motion • Understand equations of circular motion
Keywords	Circular motion, uniform circular motion, derivation of centripetal acceleration, non-uniform circular motion, equations of circular motion

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1. Unit Syllabus

Chapter 3: Motion in a straight line

Frame of reference, motion, position –time graph Speed and velocity

Elementary concepts of differentiation and integration for describing motion, uniform and non-uniform motion, average speed and instantaneous velocity, uniformly accelerated motion, velocity-time and position time graphs relations for uniformly accelerated motion - equations of motion (graphical method).

Chapter 4: Motion in a plane

Scalar and vector quantities, position and displacement vectors, general vectors and their notations, multiplication of vectors by a real number, addition and subtraction of vectors, relative velocity, unit vector, resolution of a vector in a plane, rectangular components, scalar and vector product of vectors

Motion in a plane, cases of uniform velocity and uniform acceleration projectile motion uniform circular motion.

2. MODULE-WISE DISTRIBUTION OF UNIT SYLLABUS 10 Modules

The above unit is divided into 10 modules for better understanding.

Module 1	• Introduction to moving objects • Frame of reference, • limitations of our study • treating bodies as point objects
Module 2	• Motion as change of position with time • Distance travelled unit of measurement • Displacement negative, zero and positive • Difference between distance travelled and displacement • Describing motion by position time and displacement time graphs
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Module 5	• Application of equations of motion • Graphical representation of motion • Numerical

Module 6	• Vectors • Vectors and physical quantities • Vector algebra • Relative velocity • Problems
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Module 8	• Circular motion • Uniform circular motion • Constant speed yet accelerating • Derivation of $a = \frac{v^2}{r}$ $or\ \omega^2 r$ • direction of acceleration • If the speed is not constant? • Net acceleration
Module 9	• Numerical problems on motion in two dimensions • Projectile problems
Module 10	• Differentiation and integration • Using logarithm tables

MODULE 8

3. WORDS YOU MUST KNOW

- **Frame of reference:** Any reference frame the coordinates(x, y, z), which indicate the change in position of object with time.
- **Motion in two dimension:** When the position of an object can be shown by changes any two coordinate out of the three (x, y, z), also called motion in a plane.
- **Motion in three dimension:** When the position of an object can be shown by changes in all three coordinate out of the three (x, y, z).
- **Scalar:** Physical quantity that have only magnitude.
- **Vector**: Physical quantity that has both magnitude and direction.
- **Distance travelled**: The distance an object has moved from its starting position SI unit m, this can be zero, or positive.
- **Displacement:** The distance an object has moved from its starting position moves in a particular directions S.I unit: m, this can be zero, positive or negative.
- **Path length**: Actual distance is called the path length.

- **Speed:** Rate of change of position is called speed its SI unit is m/s.

- **Velocity:** Rate of change of position in a particular direction is called velocity it can be zero, negative and positive, its SI unit is m/s.

- **Acceleration:** Rate of change of velocity is called acceleration.

- **Equations of motion:** Equations relating initial velocity, final velocity acceleration, distance travelled and the time elapsed in doing so.

- **Vector algebra:** Method of operating with vectors to add, subtract and multiply.

- **Motion in one dimension:** When a body moves in a straight line.

- **Motion in a plane**: When a body moves in a way that its position changes in 2 dimensions.

4. INTRODUCTION

You must have travelled on straight roads and highways you must also have moved on curved paths like on rounders.

When an object follows a circular path it is said to be in circular motion. However, the curved path may be a full circle or a part of a circle.

Now imagine a body moving in a circular path. The body may move with a constant speed or with variable speed.

Also, when we consider a circle- the track could be in the horizontal plane, or a vertical plane or any other plane.

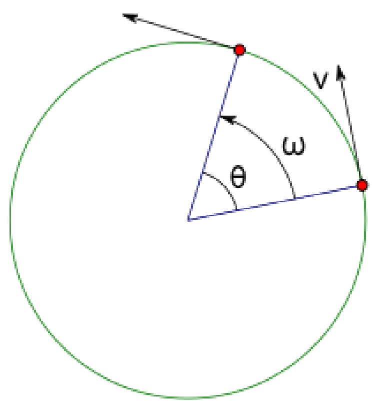
In this module we will study body moving in a circular path.

5. SPECIAL TERMS

The study of motion in a circle is unique.

Let us consider the simplest case of a body moving in a circular track with constant speed. We will study the meaning of special terms associated with circular motion

ANGULAR DISPLACEMENT



Angular displacement θ of any object is defined as the angle traced by the radius vector at the center of the circular path in the given time. i.e.

Angle = arc/radius

Or

$\Theta = \frac{arc\ length}{radius} = \frac{v}{r}$ (as shown in fig.)

It is a vector quantity and its direction depends upon sense of revolution of the body clockwise or anticlockwise. To understand this better

Consider a radius vector attached to the object, this is just a line attached to the object and it revolves around the center of the circular path. The angle it subtends at the center is the angular displacement θ

So, we use angular displacement instead of 'path length 'or displacement as the path is curved.

The SI unit of angular displacement is radian.

There is 2π radian subtended by the radius vector at the center as it completes one revolution.

ANGULAR VELOCITY

The rate of change or angular displacement is called angular velocity.

Angular velocity of a body in circular motion is defined as time rate of change of angular displacement which is denoted by the letter ω (omega) given by:

ω = angle traced/ time taken

Or $$\boldsymbol{\omega} = \frac{\textbf{angular dispalcement}}{\textbf{time elepsed}} = \frac{\Delta\theta}{\Delta t}$$

Like we had considered instantaneous linear velocity, we can define instantaneous angular velocity by choosing time interval to be as small as possible

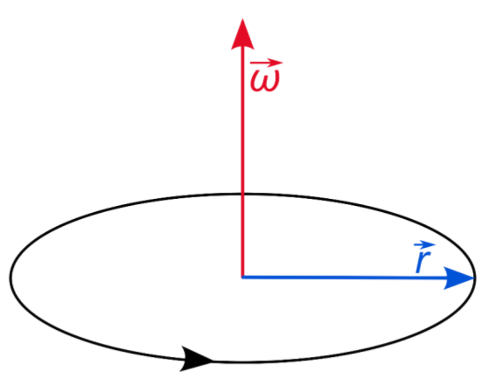
$$\textbf{instantaneous angular } \boldsymbol{\omega} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

If the object moves with constant speed the angular velocity is constant.

Angular velocity is represented by ω

its SI unit is rad/s

its dimensional formula is $[M^0L^0T^{-1}]$



This shows direction of angular velocity

PERIOD OF REVOLUTION

The time taken to complete one revolution is called period of revolution or periodic time. In case the body moves with constant speed this time is fixed. It is represented by T and SI unit is s.

We can also say

Angular velocity $\omega = \frac{2\pi}{T}$

FREQUENCY OF REVOLUTION:

For circular motion, frequency is defined as the number of revolutions completed by the body in a unit time.

It is denoted by f and SI unit s^{-1} or also called Hertz.

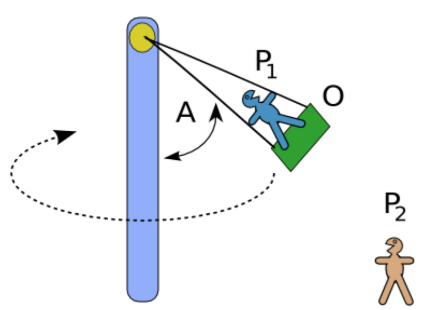
RELATION BETWEEN TIME PERIOD AND FREQUENCY:

$$\mathbf{f = \frac{1}{T}}$$

6. UNIFORM CIRCULAR MOTION

A circular motion is defined as a change in the direction of the velocity of a body or an object with or without change in its magnitude.

For example:



Here, motion of P_1 is circular motion. P_2 is an observer

The word "uniform" refers to the speed, which is uniform (constant) throughout the motion.

At constant speed, the motion of the object is called uniform circular motion.

RELATION BETWEEN SPEED AND ANGULAR VELOCITY

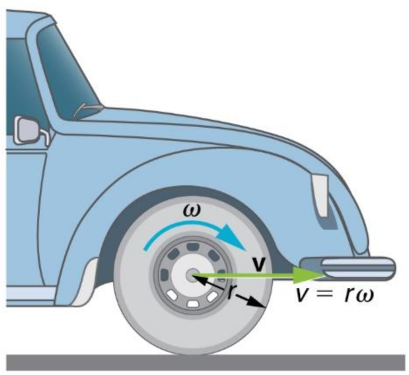
Suppose an object is moving with uniform speed v in a circle of radius R

Since the velocity of the object is changing continuously in direction, the object undergoes acceleration. Let us find the magnitude and the direction of this acceleration.

We know

$$\boldsymbol{\omega} = \frac{2\pi}{T}$$

and linear speed $v = \frac{2\pi R}{T} = \omega R$



This relationship can be illustrated by considering the tyre of a moving car, as shown in the picture. Note that the speed of the point at the center of the tyre is the same as the speed v of the car. The faster the car moves, the faster the tyres spins—large v means a large ω, because v = ω R.

Similarly, a larger-radius tyre rotating at the same angular velocity (ω) will produce a greater linear speed (v) for the car.

Boundless Physics. Boundless, 26 May. 2016. Retrieved 06 Jul. 2016 from https://www.boundless.com/physics/textbooks/boundless-physics-textbook/rotational-kinematics-angular-momentum-and-energy-9/quantities-of-rotational-kinematics-81/angular-velocity-omega-321-6953/

SOMETHING OF INTEREST

The word "uniform" refers to the speed, which is uniform (constant) throughout the motion.

https://www.youtube.com/watch?v=gpPp2gHwfRM

This video is about how would a physicist get the olive in the martini glass and perhaps slightly amuse Mr. Bond? There is much physics in this enchanting display of genius!

7. CONSTANT SPEED AND YET ACCELERATING -

Motion of an object in a circular path is a case of accelerated motion. Even if the speed is constant, since the direction of motion is changing, the velocity is not constant thus because of changing direction velocity is changing.

So we can have

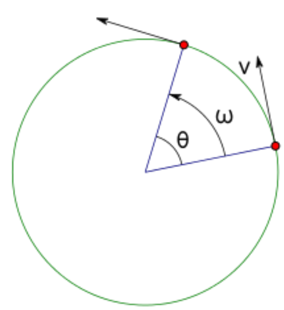
a body moving in a circle with constant speed

a body moving in a circle with variable speed

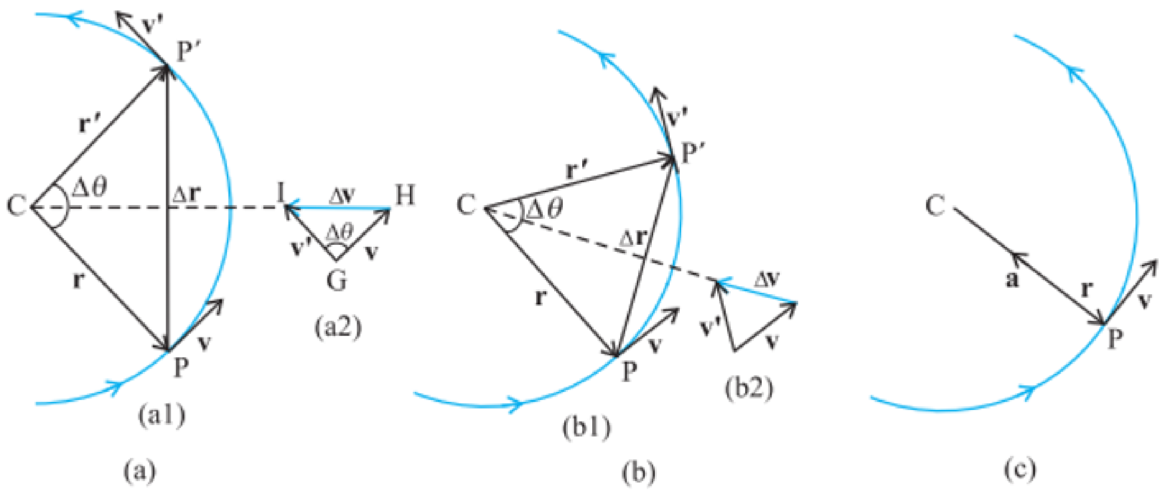
In both situations, since the direction of motion is changing the motion is accelerated

Let us consider an example:

Suppose an object is moving with uniform speed v in a circle of radius R as shown in this fig.



Since **the velocity of the object is changing continuously in direction, the object undergoes acceleration.**



Velocity and acceleration of an object in uniform circular motion. The time interval Δt decreases from (a) to (c) where it is zero. The acceleration is directed, at each point of the path, towards the centre of the circle.

DERIVATION OF a = ω^2R

Let us find the magnitude and the direction of this acceleration.

Let r and r′ be the position vectors and v and v′ the velocities of the object when it is at point P and P′ as shown in Fig.(a). By definition, velocity at a point is along the tangent at that point in the direction of motion.

The velocity vectors v and v′ are as shown in Fig. (a1). Δv is obtained in Fig. (a2)

Using the triangle law of vector addition

Since the path is circular, v is perpendicular to r and so is v′ to r′.

Therefore, Δv is perpendicular to Δr. Since average acceleration is along Δv ($\bar{a} = \frac{\Delta v}{\Delta t}$), the average acceleration $\bar{a}$ is perpendicular to Δr.

If we place Δv on the line that bisects the angle between r and r′, we see that it is directed towards the centre of the circle.

Figure (b) shows the same quantities for smaller time interval. Δv and hence $\bar{a}$ is again directed towards the centre.

In Fig.(c), $\Delta t \to 0$ and the average acceleration becomes the instantaneous acceleration. It is directed towards the centre (In the limit $\Delta t \to 0$, Δr becomes perpendicular to r. In this limit $\Delta v \to 0$ and is consequently also perpendicular to V. Therefore, the acceleration is directed towards the centre, at each point of the circular path.).

Thus, we find that the acceleration of an object in uniform circular motion is always directed towards the centre of the circle. Let us now find the magnitude of the acceleration. The magnitude of 'a' is, by definition, given by:

$$|\boldsymbol{a}| = \lim_{\Delta t \to 0} \frac{|\Delta \boldsymbol{v}|}{\Delta t}$$

Let the angle between position vectors r and r′ be $\Delta\theta$. Since the velocity vectors v and v′ are always perpendicular to the position vectors, the angle between them is also $\Delta\theta$.

Therefore, the triangle CPP′ formed by the position vectors and the triangle GHI formed by the velocity vectors v, v′ and Δv are similar (Fig. a).

Therefore, the ratio of the base-length to side-length for one of the triangles is equal to that of the other triangle. That is:

$$\frac{|\Delta \mathbf{v}|}{\mathbf{v}} = \frac{|\Delta \mathbf{r}|}{\mathbf{R}} \quad \textbf{or} \quad \Delta \mathbf{v} = \frac{\mathbf{v}|\Delta \mathbf{r}|}{\mathbf{R}}$$

Therefore,

$$|\Delta \boldsymbol{a}| = \lim_{\Delta t \to 0} \frac{|\Delta v|}{\Delta t} = \lim_{\Delta t \to 0} \frac{v|\Delta r|}{R\Delta t} = \frac{v}{R} \lim_{\Delta t \to 0} \frac{|\Delta r|}{\Delta t}$$

If Δt is small, $\Delta\theta$ will also be small and then arc PP′ can be approximately taken to be $|\Delta r|$:

$$\left|\Delta r\right| = v\Delta t$$

$$\frac{\Delta r}{\Delta t} = v$$

$$or \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = v$$

Therefore, the centripetal acceleration a_c is:

$$\mathbf{a_c} = \left(\frac{v}{R}\right)(v) = \frac{v^2}{R}$$

Thus, **the acceleration of an object moving with speed v in a circle of radius R has a magnitude v^2 /R and is always directed towards the centre. This is why this acceleration is called centripetal acceleration (a term proposed by Newton).**

A thorough analysis of centripetal acceleration was first published in 1673 by the Dutch scientist Christian Huygens (1629-1695) but it was probably known to Newton also some years earlier.

"Centripetal" comes from a Greek term which means 'centre-seeking'.

Since for a body moving at constant speed v and in a circle of radius R, v and R are both constant, hence the magnitude of the centripetal acceleration is also constant.

HOWEVER, THE DIRECTION CHANGES, Pointing always towards the centre.

Therefore, a centripetal acceleration is not a constant vector. We have another way of describing the velocity and the acceleration of an object in uniform circular motion. As the object moves from P to P′ in time Δt (= t′ – t), the line CP turns through an angle Δθ as shown in the figure above. Δθ is called angular distance.

We define the angular speed ω (Greek letter omega) as the time rate of change of angular displacement:

$$\boldsymbol{\omega} = \frac{\textbf{angular dispalcement}}{\textbf{time elepsed}} = \frac{\boldsymbol{\Delta\theta}}{\boldsymbol{\Delta t}}$$

Now, if the distance traveled by the object during the time Δt is Δs, i.e. PP′ is Δs, then:

$$v = \frac{\Delta s}{\Delta t}$$

But $\Delta s = R\ \Delta\theta$.

Therefore:

$$\mathbf{v = \omega R}$$

We can express centripetal acceleration a_c in terms of angular speed:

$$\mathbf{a_c} = \frac{v^2}{R} = \frac{\omega^2 R^2}{R} = \omega^2 R$$

As we know, the time taken by an object to make one revolution is known as its time period T and the number of revolution made in one second is called its frequency f(=1/T). However, during this time the distance moved by the object is s = 2πR.

Therefore, $v = \frac{2\pi R}{T} = 2\pi R f$

In terms of frequency f, we have

$\boldsymbol{\omega = 2\pi f}$

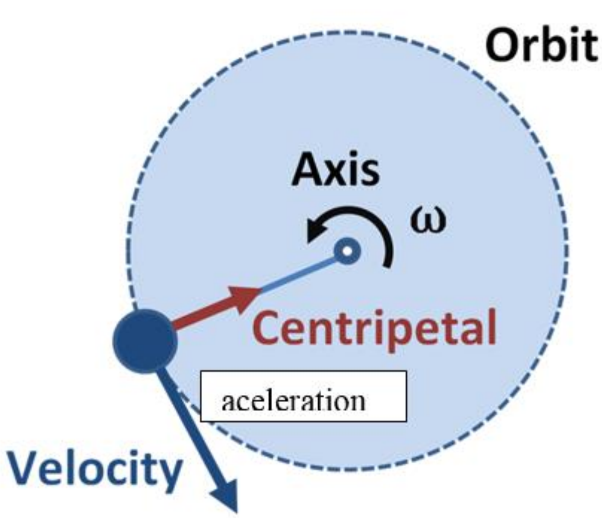
$\mathbf{v = 2\pi R f}$

$$\mathbf{a_c = 4\pi^2 f^2 R}$$

SHOWING THE DIRECTION OF CENTRIPETAL ACCELERATION:

The direction of centripetal acceleration is same as of $\Delta\vec{v}$.

Centripetal acceleration at any point in the circular path, acts along the radius of the circular path at that point and is directed towards the centre of the circular path as shown in fig. given below.



8. NON-UNIFORM CIRCULAR MOTION -ANGULAR ACCELERATION:

Suppose an object is moving with uniform speed v in a circle of radius R as shown in Fig.1 Since the velocity of the object is changing continuously in direction, the object undergoes acceleration. This as we have seen is called centripetal acceleration.

What if the speed is not constant?

ω the angular velocity will also not be constant

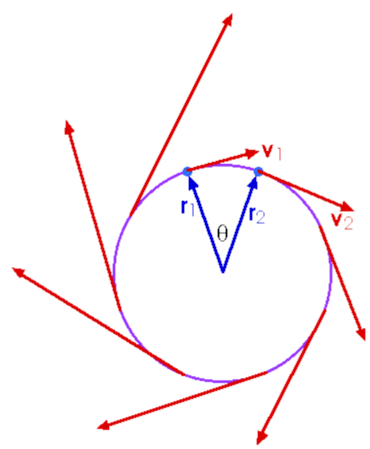
In this case there will be two accelerations

i) **Centripetal acceleration**, which acts on the body towards the center of the circle, even when the speed is constant and accounts for the continuously changing direction

ii) **Tangential acceleration and Angular acceleration**

In case **speed also changes**, the linear or tangential acceleration acts tangential to the circle, direction given on the basis of clockwise or anticlockwise revolution

tangential acceleration is a measure of how quickly a **tangential** velocity changes. It always acts perpendicular to the centripetal **acceleration** of a rotating object. It is equal to the angular **acceleration** α, times the radius of the rotation.



http://www.ux1.eiu.edu/~cfadd/1150-05/04TwoDKinematics/accel.html it is given by $a_T = \frac{\Delta v}{\Delta t}$ SI unit ms^{-2}

Angular acceleration of a body in circular motion is defined as the time rate of change of its angular velocity.

Let us find the magnitude and the direction of this acceleration

It is denoted the letter α (alpha) and given by:

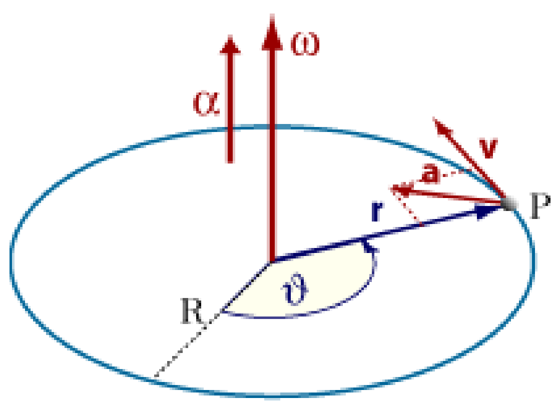
$$\alpha = \frac{\Delta\omega}{\Delta t}$$

the direction of angular acceleration is the perpendicular to the plane of the circular track and will depend on whether the object is increasing or decreasing in speed.

It is measured in radians per second per second, or $rads^{-2}$ and has dimensional formula as $[M^0 L^0 T^{-2}]$.

RELATION BETWEEN LINEAR ACCELERATION (a) AND ANGULAR ACCELERATION (α):

$a = \alpha R$ (where a= dv/dt and v = ω R)



This shows direction of angular acceleration.

Equations of motion for non-uniform circular motion with uniform angular acceleration

Equation of motion **Linear motion**	**Equation of motion** **circular motion**
v= u +at	$\omega_{f=}\omega_i + \alpha t$
$s = ut + \frac{1}{2}at^2$	$\theta = \omega_i t + \frac{1}{2}\alpha t^2$
$v^2 = u^2 + 2as$	$\omega_f^2 = \omega_i^2 + 2\alpha\theta$

9. NET ACCELERATION

In a uniform circular motion there is only centripetal acceleration, but in case of non-uniform circular motion there is tangential acceleration and acceleration perpendicular to the plane of the circle these have mutually perpendicular directions and net acceleration can be obtained by taking a vector sum.

CONCEPTUAL PROBLEMS

- **Can the direction of velocity of a body change when the acceleration is constant?**

Yes for a body moving in a circle with constant speed.

- **A stone tied to a string if whirled in a circle. If the string breaks the stone flies at a tangent. Why?**

The instantaneous velocity of the stone going around the circular path is always along the tangent to the circle. When the string breaks the stone follows the tangential path.

- **Why do cars have mud flaps?**

Loose mud, wet mud clings to the tyres if the speed of the vehicle is high the centripetal force required by mud may not be sufficient and the mud will fly off at a tangent to the tyre.

EXAMPLE

A stone tied to the end of a string of length 6 m is whirled in a circular path with constant speed. If the stone makes 10 revolutions in 20s, calculate the magnitude of acceleration.

SOLUTION

Given: r = 6m

f = no. of revolutions / time taken = 10 /20

= 0.5 rev per s

Centripetal acceleration, $a_c = r\ \omega^2$

$= 6x(2\pi f)^2$

= 59.15 m/s^2

EXAMPLE

A motor bike traveling at 50 m/s on a circular road of radius 500 m. It is increasing its speed at the rate of 2m s^{-2}. What is the net acceleration?

SOLUTION

GIVEN: v = 50 m/s

r = 500m

$a_T = 2m/s^2$

Centripetal acceleration, $a_c = \frac{v^2}{r}$ = 50 x 50 / 500 = 5m/s^2

Here, a_c is acting along the radius towards the centre of the circular path and a_T acts tangential to the circular path.

Therefore,

Effective acceleration, a = $\sqrt{a_c^2 + a_T^2} = \sqrt{5^2 + 2^2} = 5.38 ms^{-2}$

EXAMPLE

An insect trapped in a circular groove of radius 12 cm moves along the groove steadily and completes 7 revolutions in 100 s.

(a) What are the angular speed and the linear speed of the motion?

(b) Is the acceleration vector a constant vector?

What is its magnitude?

SOLUTION

This is an example of uniform circular motion.

Here R = 12 cm.

The angular speed ω is given by:

$$\omega = \frac{2\pi}{T} = 2\pi \frac{7}{100} = 0.44\, rad/s$$

The linear speed v is:

v = ω R = 0.44 s^{-1} × 12 cm = 5.3 cm s^{-1}

The direction of velocity v is along the tangent to the circle at every point.

The acceleration is directed towards the centre of the circle.

Since, this direction changes continuously, acceleration here is not a constant vector.

However, the magnitude of acceleration is constant:

a = ω^2 R = (0.44 s–1)^2 (12 cm) = 2.3 cm s^{-2}

MOTION IN A HORIZONTAL CIRCLE

This motion in a plane as the only acceleration on it is towards the center or along the tangent to the circle, effect of gravity is ignored.

TRY THESE

- **A disc revolves in a horizontal plane at a steady rate of 3 revolutions per second. A coin placed at a distance of 2 cm from the axis of rotation remains at rest on the disc. The coefficient of friction between coin and disc is**
  a. **0**.53
  b. 1.53
  c. 0.72
  d. 1.72

Answer: (c)

Hint: Let the coin of mass m be placed at r from axis of rotation, the angular velocity of coin is same as that of disc. The frictional force between coin and disc gives necessary centripetal force.

$$F = m\omega r^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

If N is normal reaction, then for vertical equilibrium

N = mg

and frictional force, $F = \mu N = \mu mg \ldots\ldots$(2)

From (1) and (2), $\mu = \frac{\omega^2 r}{g}$

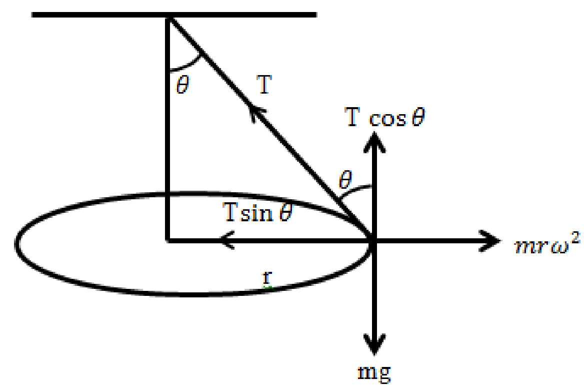
Then $\omega = 2\pi n = 6\pi\ rad/sec$

$$\boldsymbol{\mu = \frac{(6\pi)^2 \times 0.02}{9.8} = 0.72}$$

- **A heavy small sized sphere is suspended by a string of length l. The sphere rotates uniformly in a horizontal circle with the string making an angle θ with the vertical. Then the time period of this pendulum is**

    a. $T = 2\pi\sqrt{\frac{l}{g}}$
    b. $T = 2\pi\sqrt{\frac{l\sin\theta}{g}}$
    c. $T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$
    d. $T = 2\pi\sqrt{\frac{l}{g\cos\theta}}$

    Answer: (c)



Hint: Here T$\sin\theta = mr\omega^2$

$$\text{T}\cos\theta = mg$$

$$\frac{T\sin\theta}{T\cos\theta} = \frac{r\omega^2}{g}$$

Here, $\frac{r}{l} = \sin\theta$

$$\omega = \sqrt{\frac{g}{l\cos\theta}}$$

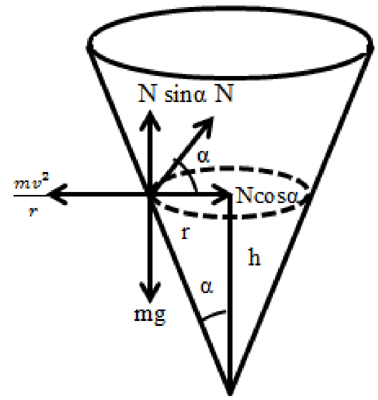
$$\therefore \boldsymbol{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l\cos\theta}{g}}}$$

- **A particle describes a horizontal circle on the smooth surface of an inverted cone, the height of plane of the circle above the vertex is 9.8 cm. The speed of the particle is**

    a. 9.8 m/sec
    b. 0.98 m/sec
    c. 1.96 m/sec
    d. 9.86 m/sec

Answer: (b)

Hint: Suppose the speed of the particle in circular motion is v, the forces acting on particle are



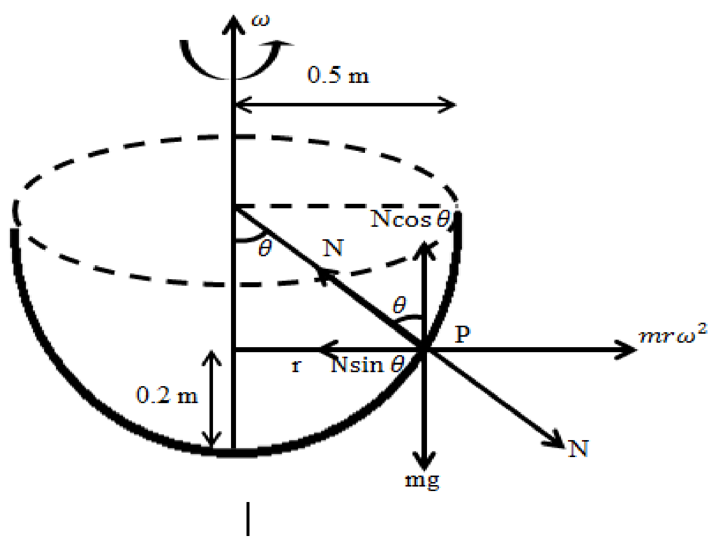
i. weight mg acting downward
ii. reaction of centripetal force $\frac{mv^2}{r}$ acting radially outward
iii. Normal reaction 'N' of smooth surface of cone which makes angle α in the horizontal.

$$\therefore N\ cos\alpha = \frac{mv^2}{r}, Nsin\alpha = mg$$

$\tan\alpha = \frac{gr}{v^2}$ But $\tan\alpha = \frac{r}{h}$

$$v = \sqrt{gh} = \sqrt{9.8 \times 9.8 \times 10^{-2}}$$
$$= \mathbf{0.98\ m/sec}$$

- **A large mass 'M' and a small mass 'm' hang at the two ends of a string that passes through a smooth tube as shown in fig. The mass 'm' moves around a circular path which lies in a horizontal plane. The length of string from the mass 'm' to the tube is l and θ is the angle this length makes with the vertical. The frequency of rotation of mass 'm' so that the bigger mass 'M' remains stationary is**



a. $n = \frac{1}{2\pi}\sqrt{\frac{Mg}{ml}}$
b. $n = \frac{1}{2\pi}\sqrt{\frac{mg}{Ml}}$
c. $n = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$
d. $n = \frac{1}{2\pi}\sqrt{\frac{Mg}{l}}$

Answer: (a)

Hint: The large mass due to its weight Mg, pulls the small mass by a force T through the string T = mg. This force T is the tension in the string at m. Due to reaction, the small mass also pulls the large mass by the force T. It is also tension in the string at M.
The mass M will remain stationary when horizontal component Tsin θ of force T will provide the necessary centripetal force mrN^2 to mass m for rotating in horizontal circle.

Here, $r = l\ sin\theta, \quad T = mg$

$$Tsin\theta = ml\ sin\theta\omega^2$$

$$T = ml\omega^2$$
$$\therefore mg = ml\omega^2$$
$$\omega = \sqrt{\frac{Mg}{ml}}$$

$$\mathbf{n = \frac{\omega}{2\pi} = 2\pi\sqrt{\frac{Mg}{ml}}}$$

- **A small mass of 10 gm lies in a hemispherical bowl of radius 0.5 m at a height of 0.2 m from the bottom of bowl. The mass will be in equilibrium if bowl rotates at an angular speed**
    a. $\frac{10}{\sqrt{3}}\ rad/sec$
    b. $10\sqrt{3}$ rad/sec
    c. 10 rad/sec
    d. $\sqrt{20}$ rad/sec

Answer: (a)

Hint: From the figure

$$\cos\theta = \frac{0.3}{0.5} = \frac{3}{5}\ and\ \sin\theta = \frac{4}{5}$$

Now, $mr\omega^2 = N sin\theta$

$$mg = N cos\theta$$

$$\omega^2 = \frac{g\tan\theta}{r} = \frac{10 \times 4}{3 \times 0.5 sin\theta}$$

From figure,

$$\frac{r}{0.5} = sin\theta$$

$$\therefore \omega^2 = \frac{10 \times 4 \times 5}{3 \times 0.5 \times 4} = \frac{100}{3}$$

$$\boldsymbol{\omega = \frac{10}{\sqrt{3}}}\ \textbf{rad / sec}$$

10. SUMMARY

In this module we have learnt

- When an object follows a circular path at constant speed, the motion of the object is called uniform circular motion.
- The magnitude of its acceleration is $a_c = v^2/R$.
- The direction of a_c is always towards the centre of the circle.

- The angular speed ω, is the rate of change of angular distance. It is related to velocity v by $v = \omega R$.
- The acceleration is $a_c = \omega^2 R$.
- If T is the time period of revolution of the object in circular motion and ν is its frequency, we have $\omega = 2\pi f$,

$v = 2\pi \nu R$,

$a_c = 4\pi^2 f^2 R$